Logistic Regression.

$$p(y=1 \mid x) = \frac{1}{1+\exp(-w^T x - b)}$$

$$p(y=0 \mid x) = 1 - p(y=1 \mid x).$$

$\therefore$ directly model the prob. of the class from data x.

$\therefore$ Discriminative model.

· $P(X)$, $P(X|y)$: Generative Method.

Bayes Rule.

$$P(y|X) = \frac{P(X|y)\,P(y)}{P(X)}$$

Probability Review.

① Random Variable.

: Maps an event to a number.

$$X : \Omega \to \mathbb{R}.$$
$$\omega \to X(\omega).$$

ex) Coin toss. (4 times)

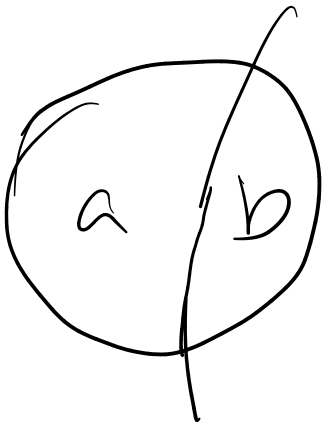
ω = "HHTH".

$X(\omega)$:= # of heads in an event ω.

$= 3$.

$$P(X=3) = \binom{4}{3} P_H^{3} (1-P_H)^{1}.$$

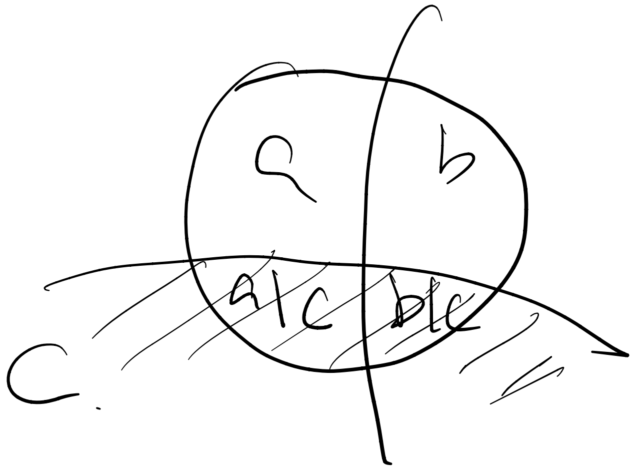
② Probability : Ratio of cardinality (area).



$$P(X=a) = \frac{A(a)}{A(a)+A(b)}$$

Conditional Proba.

$$P(X=a \mid Y=c) = \frac{A(a|c)}{A(a|c)+A(b|c)}$$



③ Probability density v.s. Probability

$p(X)$ $\mathbb{P}(X)$

i) $p(X) \geq 0$ for all $X \in \mathbb{R}$.

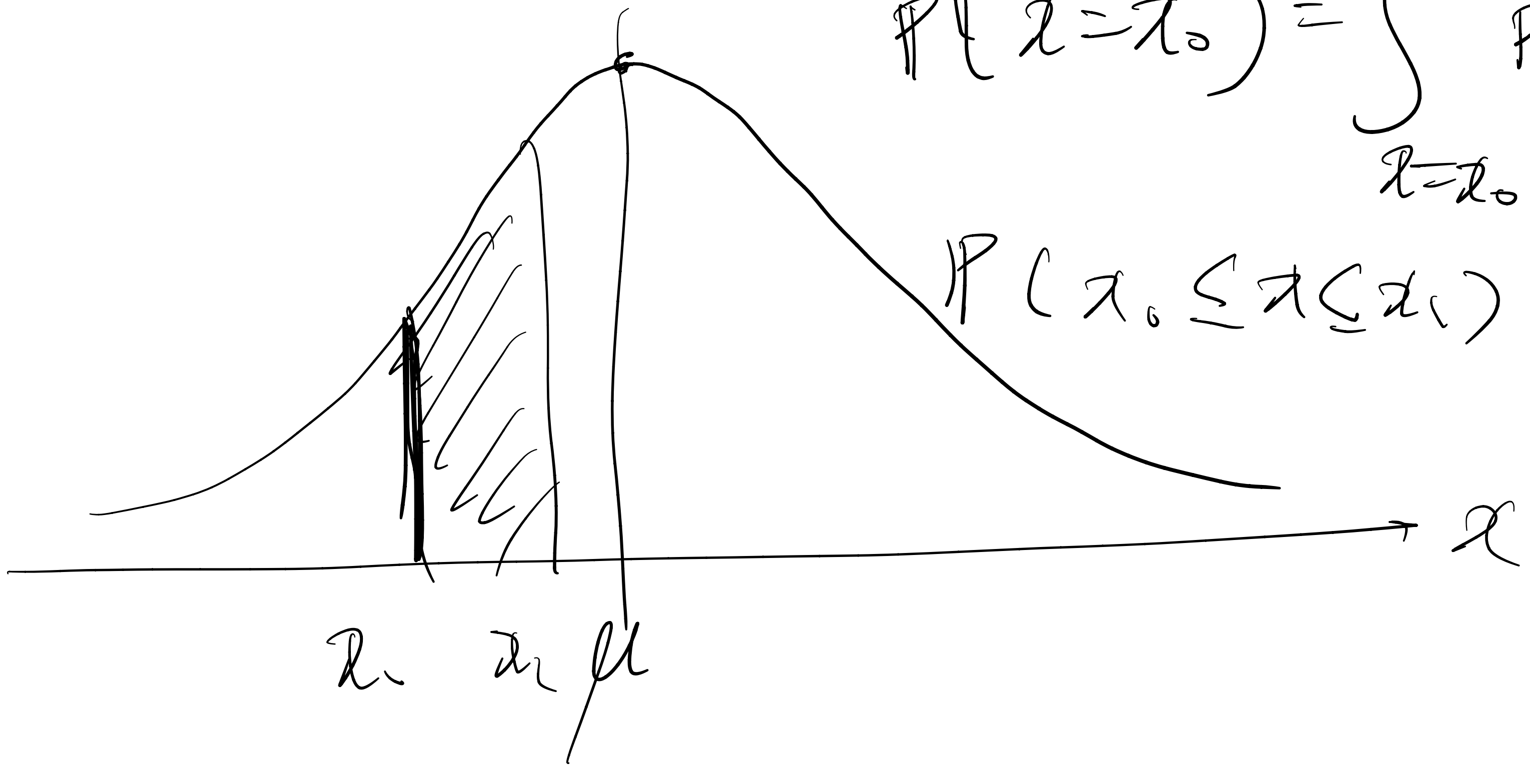
↑ Continuous variable.

ii) $\int_X p(x)\, dx = 1$.

$$\mathbb{P}(X \in Q) = \int_{X \in Q} p(x)\, dx$$

Ex). Gaussian prob. density function (p.d.f).

$$p(\underset{\sim}{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



$$\mathbb{P}(x = x_0) = \int_{x=x_0} p(x)\,dx = 0$$

$$\mathbb{P}(x_0 \leq x \leq x_1) = \int_{x_0}^{x_1} p(x)\,dx$$

④ Expectation.

$$E[f] = \langle f \rangle = \int f(x)\,p(x)\,dx.$$

- mean $\quad E[x] = \int x\,p(x)\,dx. = \langle x \rangle$

- covariance : $\quad E[\,(x - \langle x \rangle)(x - \langle x \rangle)^T\,]$

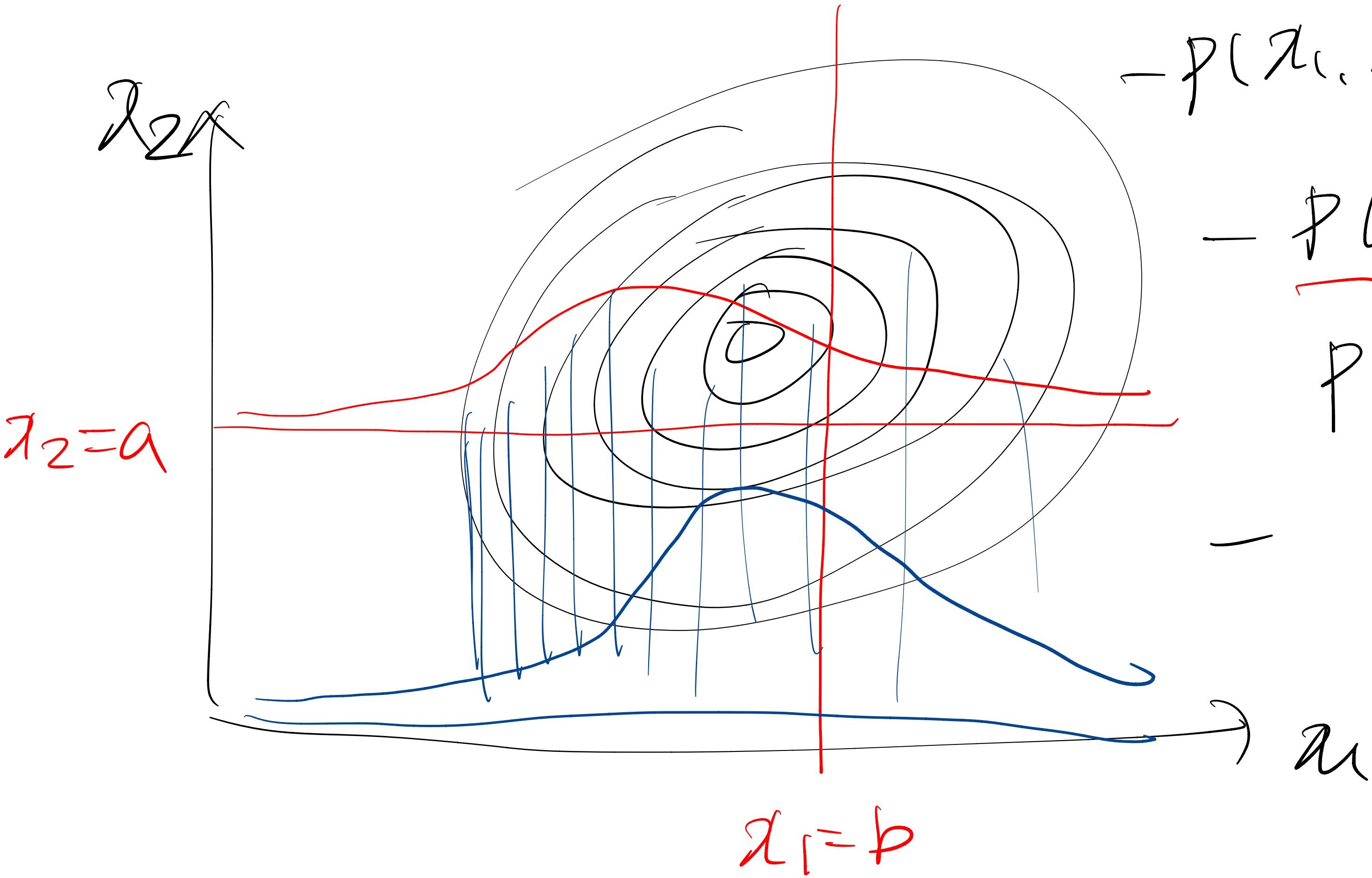
$x \in \mathbb{R}^D$.

$$= \int (x - \langle x \rangle)(x - \langle x \rangle)\,p(x)\,dx.$$

$\in \mathbb{R}^{D \times D}$

⑤ Joint P.D.F / Conditional P.D.F / Marginal P.D.F

$X \in \mathbb{R}^2$



- $p(x_1, x_2)$: Joint P.D.F.
- $p(x_1 | x_2)$

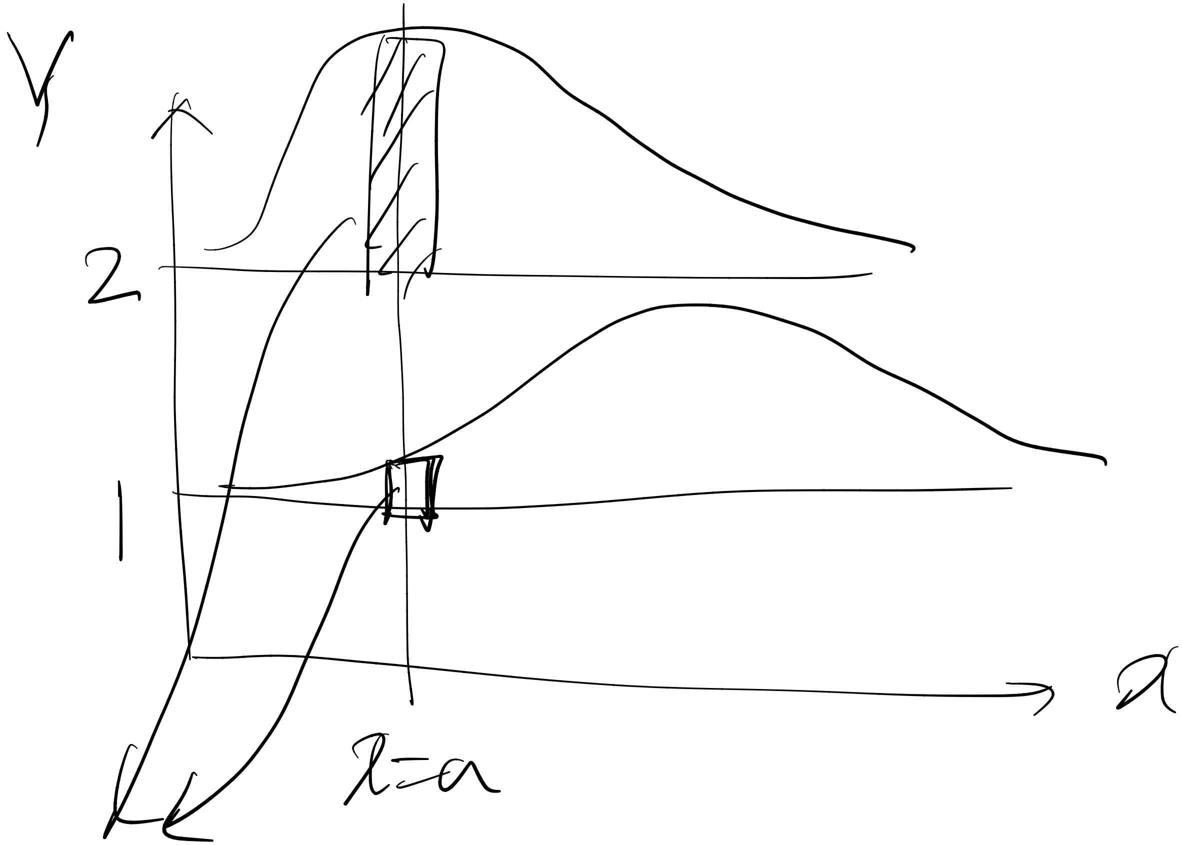
  $p(x_2 | x_1 = b)$
- $p(x_1) = \int p(x_1, x_2)\, dx_2$
- $\iint p(x_1, x_2)\, dx_1\, dx_2$

  $= 1.$

Marginalization.

$X_1 \backslash X_2$	1	2	
1	$P(X_1=1, X_2=1)$	$P(X_1=1, X_2=2)$	
2	$P(X_1=2, X_2=1)$	$P(X_1=2, X_2=2)$	
			1

$$P(X_1=1) = P(X_1=1, X_2=1) + P(X_1=1, X_2=2)$$



$P(Y \mid x=a)$

② Inference

$$P(x_1, x_2, x_3) \Rightarrow P(x_1, x_2 \mid x_3 = a)$$

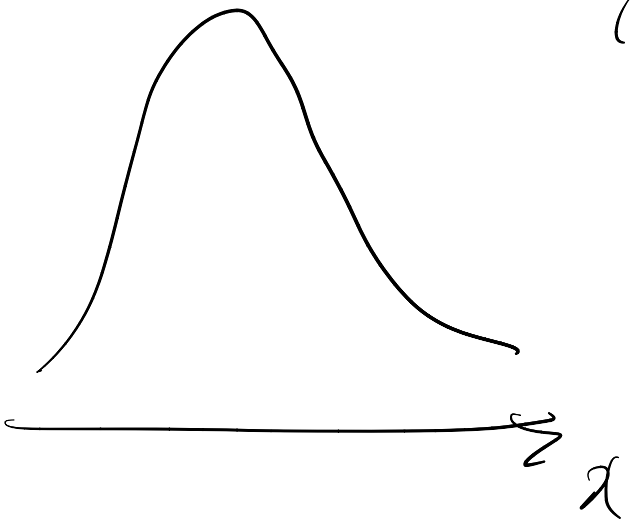
$$= \frac{P(x_1, x_2, x_3 = a)}{\boxed{P(x_3 = a)}}$$

ex) $P(x_1, x_2, x_3, y)$

age, blood, oxygen, disease (yes/no)

$$P(y \mid x_1, x_2, x_3) = \frac{\boxed{P(\quad)}}{\boxed{P(x_1, x_2, x_3)}}$$

$$\underbrace{p(x_2, x_3, y)}_{p(y \mid x_2, x_3)} = \int p(x_1, x_2, x_3, y)\, dx_1$$



i) $x \in \mathbb{R}$ $\quad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

μ: mean. σ: variance.

$p(x) \sim N(\mu, \sigma^2)$

ii) $X \in \mathbb{R}^D$ $\quad p(X) = \frac{1}{\sqrt{2\pi}^{D} \underset{\sim}{|\Sigma|^{\frac{1}{2}}}} \exp\left(-\frac{1}{2}(X-\mu)^T \Sigma^{-1} (X-\mu)\right)$

$\mu \in \mathbb{R}^D$

$\Sigma \in \mathbb{R}^{D \times D}$: symmetric p.d. matrix.