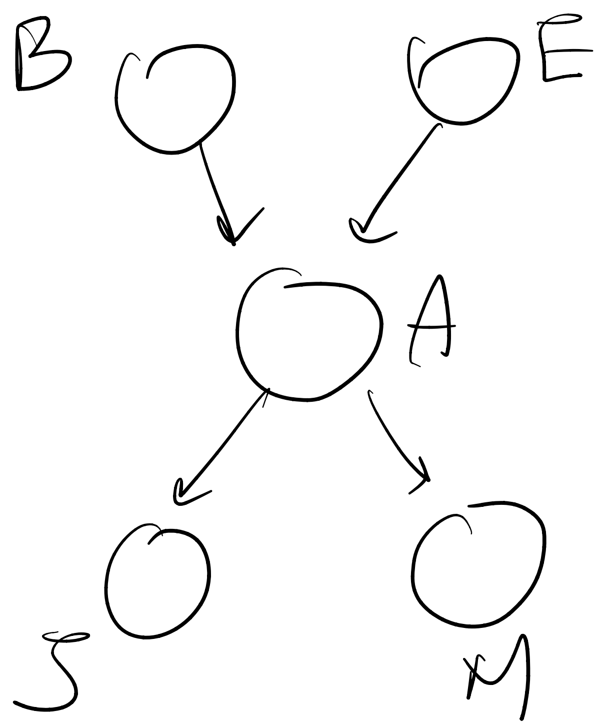
Bayesian Networks.

- DAG.
- Given a Bayesian Network, the joint p.d. can be factored as

$$p(X) = \prod_i p(x_i \mid Pa(x_i))$$

where, $pa(x_i)$: the set of parent node of x_i.

Bayesian Network: Inference.



$P(B, \neg E, A, J, \neg M)$

$P(B) = 0.05$

$P(\neg B) = 0.95$

$P(E) = 0.1$

$P(\neg E) = 0.9$

B	E	P(A|B, E)
0	0	0.05
0	1	0.5
1	0	0.85
1	1	0.95

A	P(J=1|A)
0	0.05
1	0.7

A	P(M=1|A)
0	0.15
1	0.8

① $P(B, \neg E, A, J, \neg M)$

$= P(B) \cdot P(\neg E) \cdot \underline{P(A \mid B, \neg E)}\, P(J \mid A)\, P(\neg M \mid A)$

$= 0.05 \times 0.9 \times 0.85 \times 0.7 \times 0.2 = \boxed{}$

② $P(B \mid J, \neg M)$

$$= \frac{P(B, J, \neg M)}{P(J, \neg M, B) + P(J, \neg M, \neg B)}$$

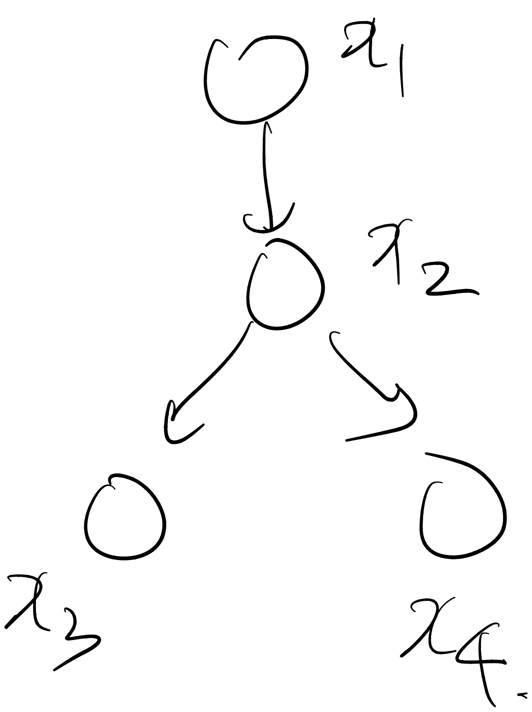
i) $P(B, J, \neg M) = \sum_{A} \sum_{E} P(B, J, \neg M, A, E)$

$=$

$$= P(B, J, \neg M, A, E) + P(B, J, \neg M, \neg A, E)$$

$$+ P(B, J, \neg M, A, \neg E) + P(B, J, \neg M, \neg A, \neg E)$$

$$c)\ P(J, \neg M, \neg B) = \sum_{A} \sum_{E} P(J, \neg M, \neg B, A, E)$$



$x_1, x_2, x_3, x_4 \in \{0, 1\}$.

$P(x_1 = 0) = 0.2$

$P(X_2 = 0 \mid X_1 = 0) = 0.4$

$P(X_2 = 0 \mid X_1 = 1) = 0.7$

$P(X_3 = 0 \mid X_2 = 0) = 0.1$

$P(X_3 = 0 \mid X_2 = 1) = 0.4$

$P(X_4 = 0 \mid X_2 = 0) = 0.2$

$P(X_4 = 0 \mid X_2 = 1) = 0.3$.

① $P(X_3=1)$

$$= \sum_{X_1}\sum_{X_2}\sum_{X_4} P(X_1, X_2, X_3=1, X_4)$$

$$= \sum_{X_1}\sum_{X_2}\sum_{X_4} P(X_1)\,P(X_2|X_1)\,P(\overset{=1}{X_3}|X_2)\,P(X_4|X_2)$$

$$\left(\sum_{X_4} P(X_4|X_2) = 1\right)$$

$$= \sum_{X_1}\sum_{X_2} P(X_1)\,P(X_2|X_1)\,P(X_3=1\,|X_2)$$

$$
\begin{aligned}
&= P(X_1=0)\,P(X_2=0 \mid X_1=0)\,P(X_3=1 \mid X_2=0) \\
&+ P(X_1=0)\,P(X_2=1 \mid X_1=0)\,P(X_3=1 \mid X_2=1) \\
&+ P(X_1=1)\,P(X_2=0 \mid X_1=1)\,P(X_3=1 \mid X_2=0) \\
&+ P(X_1=1)\,P(X_2=1 \mid X_1=1)\,P(X_3=1 \mid X_2=1)
\end{aligned}
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