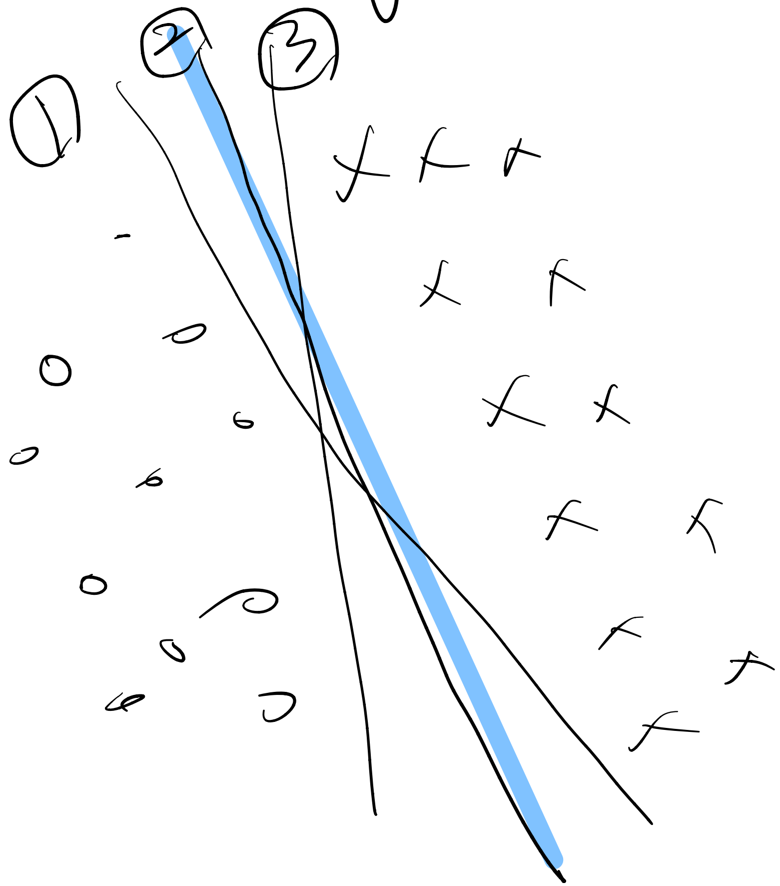
Large Margin Classifier

① Linearly Separable setting.

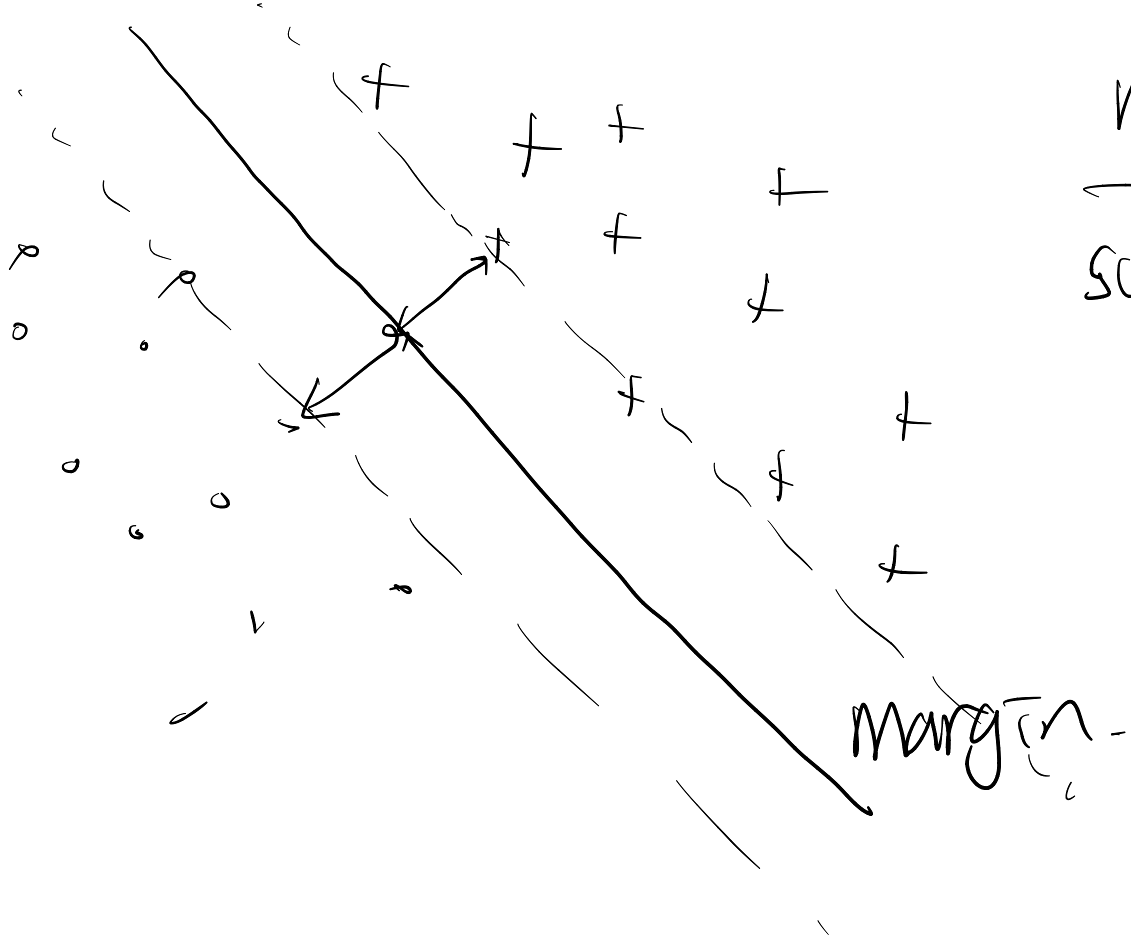
$X \in \mathbb{R}^D$



o : class 1

x : class −1.

$y \in \{1, -1\}$.

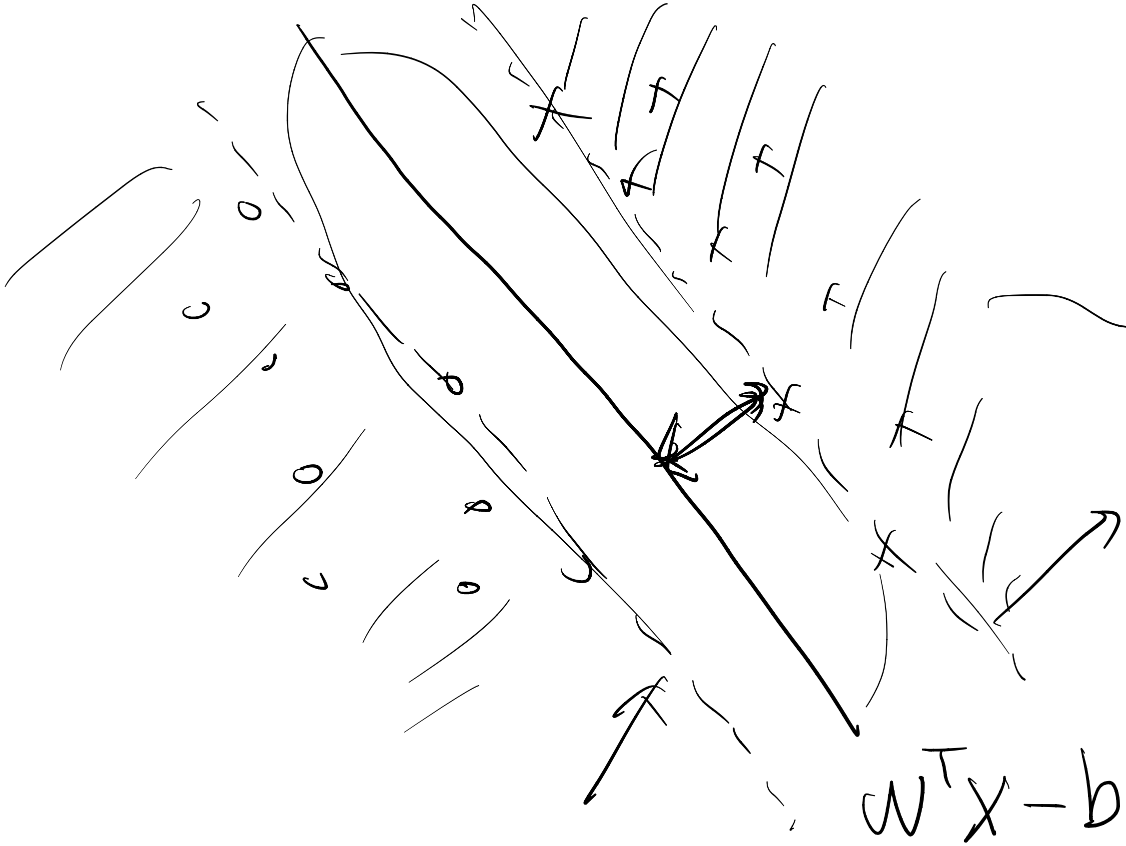


maximize margin

subject to

all data are classified correcty

$$\begin{cases} W^T X - b > 1 & \text{for } y=1 \\ W^T X - b < -1 & \text{for } y=-1 \end{cases}$$



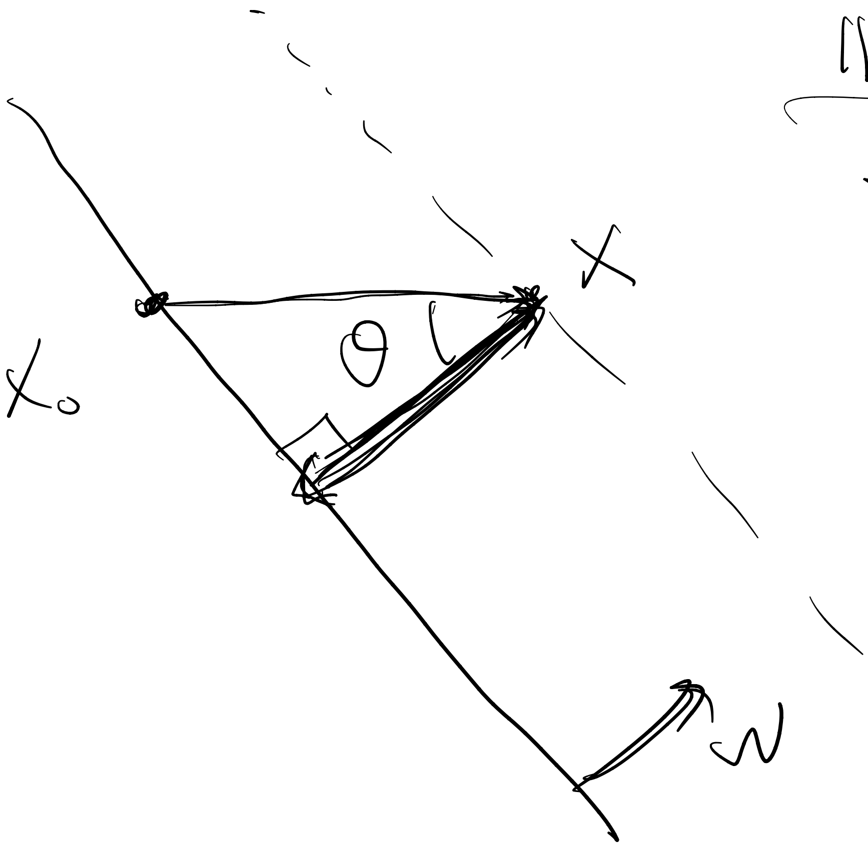
$W^T X - b = -1$

$W^T X - b = 0$

$W^T X - b = 1$

$W^T X_{new} - b > 0$

< 0



$$w^T x - b = 1$$

$$w^T x - b = 0$$

$$\|w\| \, \|x - x_0\| \cos\theta$$

$$= \|w\| \, \|x_u\|$$

$$= w^T (x - x_0) = w^T x - w^T x_0$$

$$= 1$$

$$\|w\| \, \|x_u\| = 1$$

$$\|x_u\| = \frac{1}{\|w\|}$$

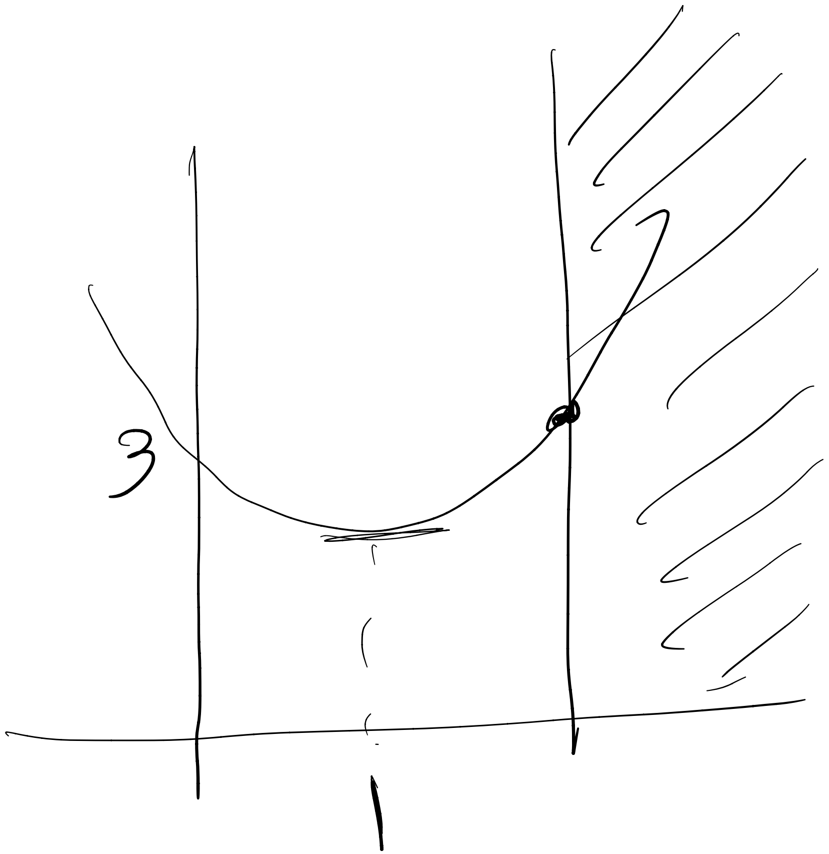
Large Margin Classifier.

$$\left(\max \frac{1}{\|W\|}\right) \quad \min \|W\|^2 \quad \text{s.t.} \quad W^T x_i - b > 1 \text{ for } y_i = 1$$
$$W^T x_i - b \leq -1 \text{ for } y_i = -1$$

$$\Rightarrow \boxed{\min \|W\|^2 \quad \text{s.t.} \quad y_i (W^T x_i - b) \geq 1 \quad \text{for all } i \text{ in } \{1, \cdots, N\}.}$$

Optimization with constraints.

$y = x^2 - 2x + 3$, $x \geq 3$. Find the min y.



feasible region.

optimize $L(x)$ with a constraint $g(x)$

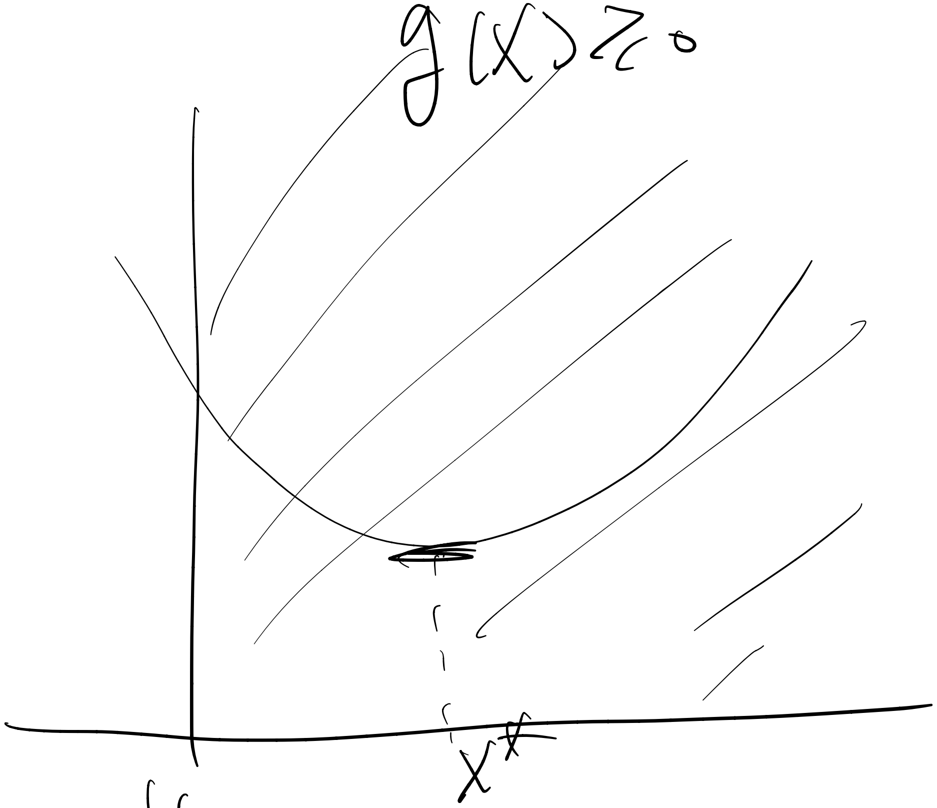
$$\max_{x} L(x) \quad s.t. \quad g(x) \geq 0$$

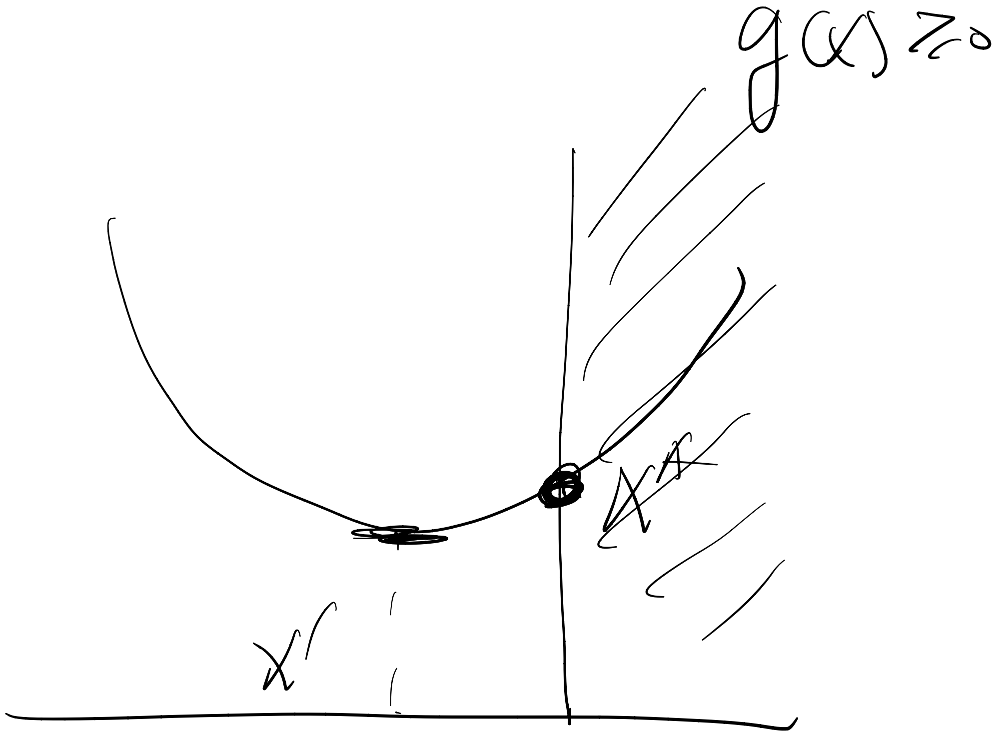
$$L'(x) = L(x) + \lambda g(x)$$

$$\frac{dL'(x)}{dx} = 0 .$$

$$\frac{dL'(x)}{dx} = \frac{dL}{dx} + \lambda \frac{dg}{dx} = 0 .$$

$$\begin{cases} \left.\dfrac{dL}{dx}\right|_{x=x^*} = 0 \\ \lambda = 0 , \quad g(x^*) \geq 0 \\ \hline \left.\dfrac{dL}{dx}\right|_{x=x'} = 0 \\ \lambda > 0 , \quad g(x^*) = 0 \end{cases}$$



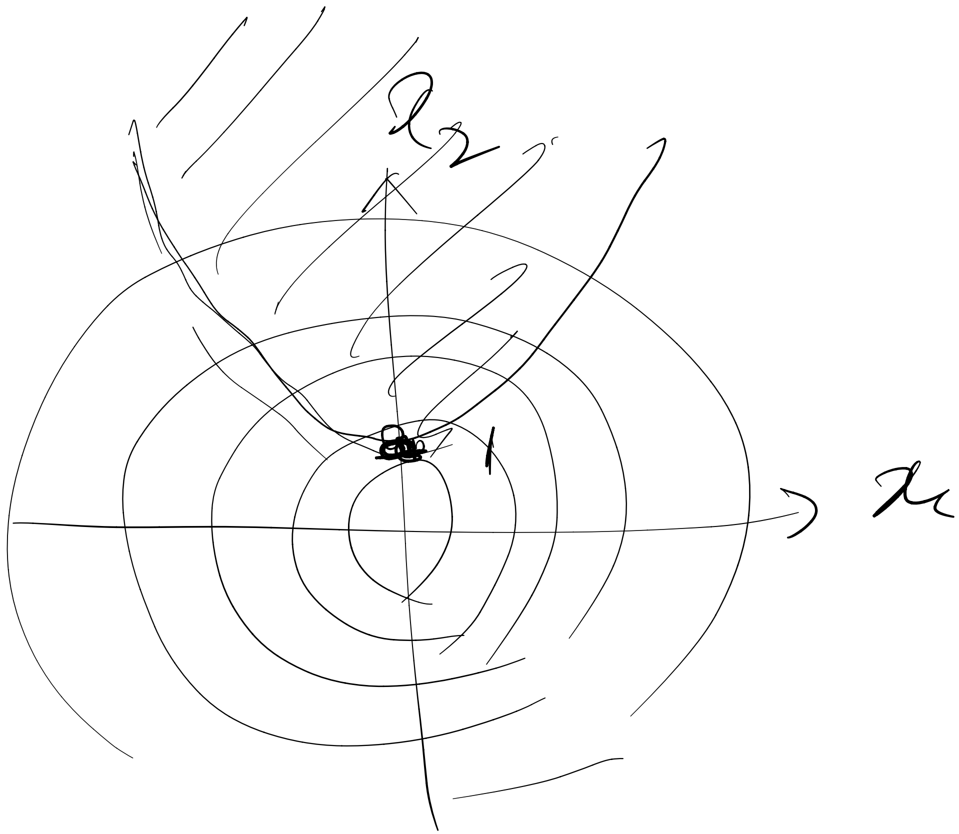


$$\begin{cases} \left.\dfrac{dL}{dx}\right|_{x=x^*} = 0 \\ \lambda = 0 \\ g(x^*) \geq 0 \end{cases}$$

$$\begin{cases} \left.\dfrac{dL}{dx}\right|_{x=x'} = 0 \\ \lambda > 0 \\ g(x^*) = 0 \end{cases}$$

$$\min_{x} L(x) = x_1^2 + x_2^2 \qquad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$g(x) = x_2 - x_1^2 - 1 \geq 0$$



$$x_2 \geq x_1^2 + 1$$

$$L'(x) = L(x) - \lambda g(x).$$

$$= x_1^2 + x_2^2 - \lambda(x_2 - x_1^2 - 1)$$

$$\boxed{\frac{\partial L'}{\partial x_1} = 2x_1 - \lambda(-2x_1) = 0 \qquad \frac{\partial L'}{\partial x_2} = 2x_2 - \lambda = 0.}$$

If $\lambda = 0$

$x_1 = 0,\ x_2 = 0$

$g(x_1 = 0, x_2 = 0) = -1 \neq 0$

$\lambda > 0 : g(x) = 0.$

$x_2 - x_1^2 - 1 = 0$

$\lambda = 2x_2$

$2(1+\lambda)x_1 = 0.$

$\Rightarrow \boxed{x_1 = 0,\ x_2 = 1}$