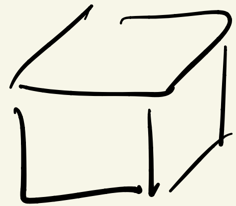
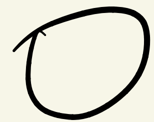
Multinomial Dist. (Binomial)





① coin toss

outcomes : H, T.

R.V : m_1, m_2 : # of H events
T events.

$$P(m_1=3, m_2=4) = \frac{7!}{3!\,4!}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^4$$

$$P(m_1, m_2, \cdots, m_k \mid \underbrace{\mu_1, \mu_2, \cdots, \mu_k}_{K})$$

$$= \binom{N}{m_1, m_2, \cdots, m_k} \prod_{k=1}^{K} \mu_k^{m_k}$$

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$$(1, 1, 2, 2, 4, 4, 4) \Rightarrow \boxed{\left(\frac{1}{12}\right)^2 \left(\frac{1}{6}\right)^2 \left(\frac{1}{3}\right)^3}$$

$$P(m_1=2, m_2=2, m_4=3) = \frac{7!}{2!\,2!\,3!} \left(\frac{1}{12}\right)^2 \left(\frac{1}{6}\right)^2 \left(\frac{1}{3}\right)^3$$

among N events,

$m_1, m_2, \cdots, m_K \Rightarrow \mu_1, \cdots, \mu_K.$

$$\underset{\mu_1 \cdots \mu_K}{\operatorname{argmax}} \left[\ln P(m_1, m_2, \cdots m_K \mid \mu_1, \cdots, \mu_K) + \lambda \left(\sum_{k=1}^{K} \mu_k - 1 \right) \right]$$

$$L = \ln \frac{N!}{m_1! \cdots m_K!} \prod_{k=1}^{K} \mu_k^{m_k} + \lambda \left(\sum_{k=1}^{K} \mu_k - 1 \right)$$

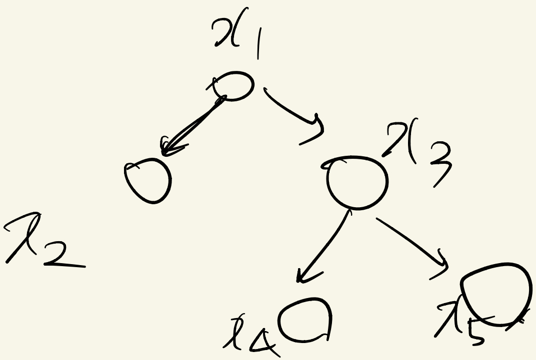
$$\frac{dL}{d\mu_k} = \frac{d}{d\mu_k} \left(\sum_k m_k \mu_k + \lambda \left(\sum_{k=1}^{K} \mu_k - 1 \right) \right)$$

$$= m_k \cdot \frac{1}{\mu_k} + \lambda = 0 \qquad \therefore \mu_k = -\frac{m_k}{\lambda}.$$

$$\sum_{k=1}^{K} \mu_k = -\sum_{k=1}^{K} \frac{m_k}{\lambda} = 1 \qquad \lambda = -\sum_k m_k = -N$$

$$\therefore \quad \hat{\mu}_k = \frac{m_k}{N},$$

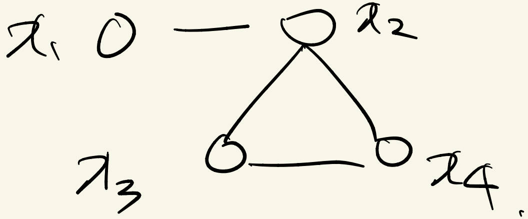
Graphical Models $\begin{cases} \text{Directed Acyclic Graph.} \\ \quad \rightarrow \text{Bayesian Network.} \\ \text{Undirected Graph} \\ \quad \rightarrow \text{Markov Random Field.} \end{cases}$



$$p(x) = p(x_1, x_2, x_3, x_4, x_5)$$

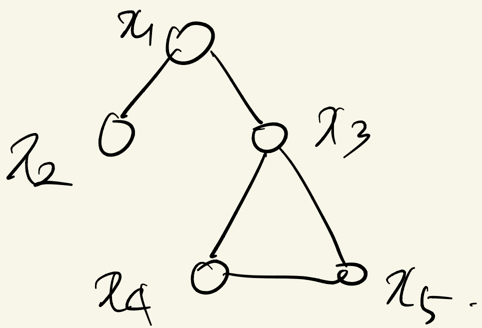
$$= p(x_1)\,p(x_2|x_1)\,p(x_3|x_1)$$

$$p(x_4|x_3)\,p(x_5|x_3).$$



$$P(X) = P(x_1, x_2, x_3, x_4)$$

Markov Random Field (MRF)



① direct edge btw x_i & x_j

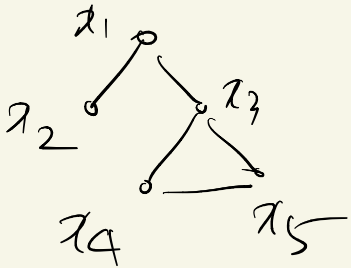
$\Rightarrow x_i \not\perp\!\!\!\perp x_j \mid x_{\setminus i,j}$

② no diret edge btw x_i & x_j

$\Rightarrow x_i \perp\!\!\!\perp x_j \mid x_{\setminus i,j}$

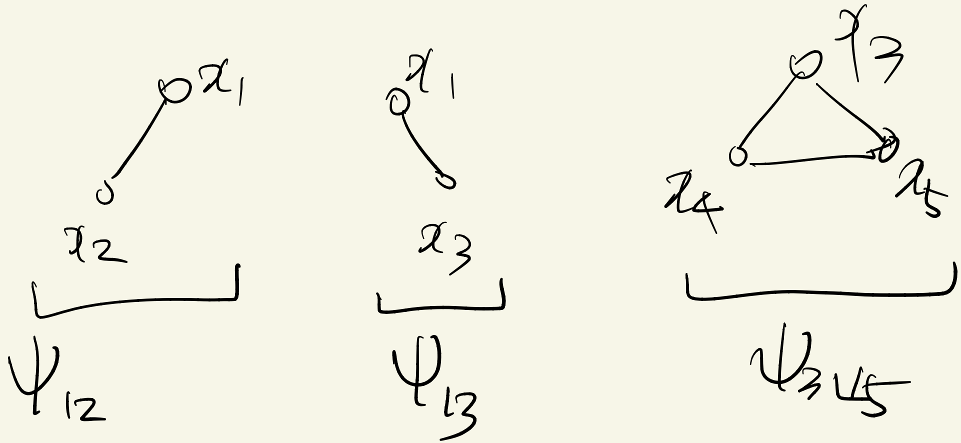
$x_1 \not\perp\!\!\!\perp x_3 \mid x_2, x_4, x_5$

$x_1 \perp\!\!\!\perp x_5 \mid x_3$



"Find maximal cliques"

= Completed graphs.



$$P(x_1, x_2, x_3, x_4, x_5) = \frac{1}{Z} \psi_{12} \psi_{13} \psi_{345}$$

$$Z = \sum_{x_1, \ldots, x_5} \psi_{12}\, \psi_{13}\, \psi_{345}.$$

Hammersly - Clifford Theorem.