Review 4−2

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Problem 1

Guess the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + cn$, where c is constant, is $\Theta(n \lg n) by applealing to a recursion tree.$

Solution 1

- level 0
  $\Sigma = cn$
- level 1
  $\Sigma = cn/3 + 2cn/3 = cn$
- level 2
  $\Sigma = cn/9 + 2cn/9 + 2cn/9 + 4cn/9 = cn$
- level k
  $\Sigma = cn$
- level h
  $\Sigma = cn$

The shortest depth $n \to 1$ is $h = \lg_{3/2} n$
The longest depth $n \to 1$ is $h = \lg_3 n$

so $T(n) = \text{depth} * cn$ and

$cn \lg_{3/2} n \leq T(n) \leq cn \lg_3 n$

therefore $T(n) = \Theta(n \lg n)$