Review 3

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Problem 1

Write `O` if an entry is true or `X` otherwise.

Solution 1

	$O(n \lg n)$	$\Omega(n \lg n)$	$\Theta(n \lg n)$
$\lg n$	O	X	X
n	O	X	X
$n \lg n$	O	O	O
$n \lg^2 n$	X	O	X
n^2	X	O	X

Problem 2

Show $3n + 1 = O(n^2)$ by the definition of O.

Solution 2

A function $f(n) = O(g(n))$ if there exist constants $c \geq 0$ and $n_0 \geq 0$, s.t.

$$n \geq n_0 \Rightarrow \leq |f(n)| \leq c|g(n)|$$

let $g(n) = n^2$
let $f(n) = 3n + 1$

suppose $c = 4,\ n_0 = 1$

and then, for all $n \geq 1 \rightarrow |3n+1| \leq 4n^2$

therefore, $3n + 1 = O(n^2)$ by the above definition.

Problem 3

Write asymptotic notations that satisfy each relation and explain why.

1. Transitivity
2. Reflexivity
3. Symmetry

Solution 3

1. Transitivity

- O is transitive
  because $f(n) = O(g(n))$ and $g(n) = O(h(n))$ implies $f(n) = O(h(n))$
  there must exists $n_0 \geq 0$, s.t. $n \geq n_0 \Rightarrow f(n) \leq c_0 g(n) \leq c_1 c_0 h(n)$
- Ω is transitive
  because $f(n) = \Omega(g(n))$ and $g(n) = \Omega(h(n))$ implies $f(n) = \Omega(h(n))$
  there must exists $n_0 \geq 0$, s.t. $n \geq n_0 \Rightarrow f(n) \geq c_0 g(n) \geq c_1 c_0 h(n)$
- Θ is transitive
  because $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$ implies $f(n) = \Theta(h(n))$

$$f(n) = O(h(n)) \wedge f(n) = \Omega(h(n))$$

2. Reflexivity

- O is reflexive
  because $f(n) = O(f(n))$ where $c = 1$
- Ω is reflexive
  because $f(n) = \Omega(f(n))$ where $c = 1$
- Θ is reflexive

- because $f(n) = \Theta(f(n))$

3. Symmetry

- O is **not** symmetric
  because $f(n) = O(g(n))$ does not imply $g(n) = O(g(n))$
  for example, $n = O(n^2)$ cannot imply $n^2 = O(n)$
- Ω is **not** symmetric
  because $f(n) = \Omega(g(n))$ does not imply $g(n) = \Omega(g(n))$
  for example, $n^2 = \Omega(n)$ cannot imply $n = \Omega(n^2)$
- Θ is symmetric
  because $f(n) = \Theta(g(n))$ implies $g(n) = \Theta(g(n))$